Collective excitations of a trapped Bose-Einstein condensate in the presence of a 1D optical lattice

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We study low-lying collective modes of a horizontally elongated ^{87}Rb condensate produced in a 3D magnetic harmonic trap with the addition of a 1D periodic potential which is provided by a laser standing-wave along the horizontal axis. While the transverse breathing mode results unperturbed, quadrupole and dipole oscillations along the optical lattice are strongly modified. Precise measurements of the collective mode frequencies at different height of the optical barriers provide a stringent test of the theoretical model recently introduced [M. Krämer *et al.* Phys. Rev. Lett. **88** 180404 (2002)].

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The measurement of frequencies of collective modes was immediately recognized to be a fundamental and precise tool to investigate the quantum macroscopic behaviour of atomic Bose-Einstein condensates. The observations of low-lying excitations [1, 2, 3], including the scissor mode [4], were an important step toward the characterization of these systems. Collective excitations were also studied at finite temperature [5, 6, 7, 8] investigating frequency shifts and damping rates. All the experiments so far reported were performed on Bose-Einstein condensates magnetically trapped in pure harmonic potentials.

More recently, condensates trapped in periodic potentials have attracted much interest demonstrated by the flourishing of both theoretical and experimental papers in this field. BEC trapped in a periodic potential is particularly interesting as a possible system for the investigation of pure quantum effects. Many results have been reported e.g. interference of atomic de Broglie waves tunneling from a vertical array of BECs [9], observation of squeezed states in a BEC [10], realization of a linear array of Josephson junctions [11] with BECs, observation of Bloch oscillations [12] and culminating with the observation of the quantum phase transition from a superfluid to a Mott insulator [13]. Recently, the general interest of this field has been extended to a careful study of the loading of BECs in an optical lattice [14] and to the observation of collapse and revival of the matter wave field of a BEC [15]. In this context, the characterization of collective modes of a condensate in the presence of an optical lattice motivated the development of a new theoretical treatment [16] based on a generalization of the hydrodynamic equation of superfluids for a weakly interacting Bose gas [17, 18] to include the effects of the periodic potential. The collective modes of a trapped BEC are predicted to be significantly modified by the presence of an optical lattice. Precise measurements of the low-lying collective frequency would verify the validity of the mass renormalization theory [16].

In this Letter we quantitatively investigate the modification of the low-lying excitation spectrum of a harmonically trapped BEC, due to the presence of a 1D optical lattice. In particular we measure the frequencies of the quadrupole and transverse breathing modes as a function of the optical lattice depth. The experimental observations are compared with the predictions of the model reported in [16] consisting in a frequency shift of the collective modes characterized by a motion along the lattice axis, whereas the frequency of collective modes involving an atomic flow transverse respect to the periodic potential, is expected to remain unchanged.

The experiment is performed with a $F = 1, m_F = -1$ ^{87}Rb condensate produced in the combined potential obtained superimposing a 1D optical lattice along the axial direction of a Ioffe-Pritchard magnetic trap [11, 19]. The harmonic magnetic trap is characterized by a cylindrical symmetry with axial and radial frequencies respectively $\nu_z = 8.70 \pm 0.02$ Hz and $\nu_\perp = 85.7 \pm 0.6$ Hz. The 1D optical periodic potential is provided by retroreflecting along the axial direction a far detuned laser beam. The light is produced by a commercial Ti:Sapphire laser tuned at $\lambda = 757$ nm. The radial dimension of the laser beam in correspondence of the condensate is ~ 300 μm, large enough, compared to the radial size of the condensate, to neglect the effect of the optical potential on the radial dynamics. The optical potential height V_{opt} has been varied in the experiment up to 5.2 E_r, $E_r = h^2/2m\lambda^2$ being the recoil energy of an atom of mass m absorbing one lattice photon. The corresponding photon scattering rate, below 0.04 s^{-1}, is negligible in the time scale of our experiment. In the experiment the lattice depth spans from the weak- to the tight-binding regime in order to investigate the generality of the theoretical predictions in [16].

After the condensate has been produced, in order to excite the collective modes, we perturb the magnetic bias field [1, 2, 3] by applying five cycles of resonant sinusoidal modulation, thus producing a periodical perturbation of the radial frequency of the magnetic trap. We modulate the radial frequency by 10% of its value. A bigger oscillation amplitude would produce a loss of superfluidity caused by entering regions of unstable dynamics [20]. This procedure excites modes with zero angular momen-

tum along the symmetry axis of the trap. For an elliptical trap, the two lowest energy modes of this type correspond to in-phase oscillations of the width along x and y and out-of-phase along z (quadrupole mode), and an in-phase compressional mode along all directions (breathing mode). In the Thomas-Fermi regime, for small amplitude oscillations and strongly elongated traps, the two modes are characterized respectively by the frequencies $\sqrt{5/2}\nu_z$ and $2\nu_\perp$ [17]. In this limit the two frequencies are quite different and the axial and radial excitations are almost decoupled. The axial width performs small amplitude oscillations at $2\nu_\perp$ superimposed to wider amplitude oscillations at frequency $\sqrt{5/2}\nu_z$ (quadrupole mode), and vice versa for the radial width (transverse breathing mode).

After exciting the collective mode, we let the cloud evolve for a variable time t, then we switch off the combined trap, let the cloud expand for 29 ms and take an absorption image of the expanded cloud along one of the radial directions. In the regime of large optical lattice heights the density profile after the expansion results in an interferogram consisting of a central cloud and two lateral peaks [19]. From the image we extract the radial ($R_\perp$) and axial (R_z) radii of the central peak as a function of time t. In order to compensate the effect of fluctuations in the condensates number of atoms we then plot the aspect ratio $R_\perp/R_z$ and fit the data to obtain the mode frequency.

Quadrupole Mode. Following [16], a trapped BEC in the presence of a 1D optical lattice can still be described by the hydrodynamic equations that take the same form as in the absence of the lattice once defined a renormalized interaction coupling constant and an effective mass m^*. The effective mass depends on the tunneling rate between adjacent optical wells thus accounting for the modified inertia of the system along the lattice. In particular, in the linear regime of small amplitude oscillations, the new frequency of the collective modes is simply obtained by replacing the axial magnetic trap frequency ν_z with $\nu_z\sqrt{m/m^*}$. In our experimental configuration, where we have an elongated magnetic trap ($\nu_z \ll \nu_\perp$), the dipole mode along the periodic potential and the quadrupole mode are both shifted and characterized respectively by the frequencies

$$\nu_D = \sqrt{\frac{m}{m^*}}\nu_z \qquad (1)$$
$$\nu_Q = \sqrt{\frac{5}{2}}\sqrt{\frac{m}{m^*}}\nu_z = \sqrt{\frac{5}{2}}\nu_D$$

To resonantly excite the quadrupole mode we thus need to first measure ν_D. The measurement of the dipole mode frequency is done as in [11] where we already investigated the dependence of the frequency of the dipole mode in conjunction with current/phase dynamics in an array of Josephson junctions. In particular, we induce dipole oscillations, by suddenly displacing the position of the magnetic field minimum along z and observing the center-of-mass motion as a function of time. The quadrupole mode

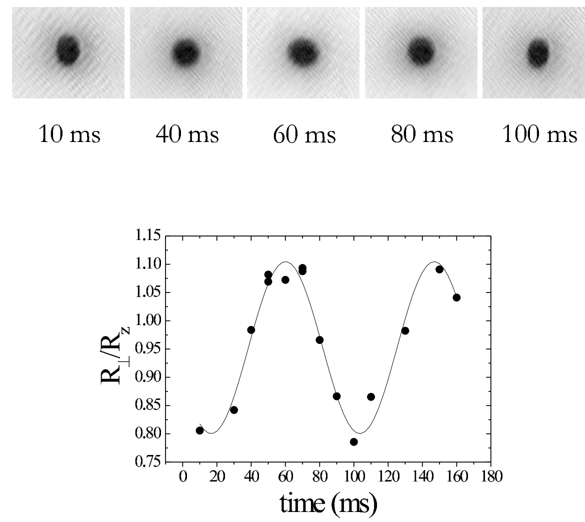

FIG. 1: In the upper part we show absorption images taken after exciting the quadrupole mode of a condensate in the combined trap with $V_{opt} = 3.4\ E_r$ and waiting different evolution times (10,40,60,80 and 100 ms) before switching off the trap and letting the cloud expand for 29 ms. In the lower part we show the evolution of the Aspect Ratio ($R_\perp/R_z$) of the central interference peak obtained from the absorption images together with a sinusoidal fit to extract the frequency of the mode.

is then excited by modulating the magnetic bias field at a frequency close to $\sqrt{5/2}\nu_D$. The procedure to excite the quadrupole mode takes ~700 ms producing a significant heating of the condensate. The final observed temperature of $\sim$ 150 nK (0.9 T_c) is consistent with the heating rate of 64 ± 7 nK/s measured in the absence of the optical lattice. The heating results in a degradation of the interference pattern visibility so that typically only the central peak is observable also for our larger laser intensities.

A typical series of data is represented in Fig. 1 where, in the upper part, we show images of the expanded condensate taken at different times after the excitation procedure and in the lower part we report the measured aspect ratio together with the sinusoidal fit. We repeated the same procedure for various intensities of the laser light creating the optical lattice. Even if the data were taken at finite temperature, we do not observe any significant damping rate in the time scale of our experiment where we follow the oscillation for $\sim$ 200 ms. This is consistent with previous results for the quadrupole mode in pure magnetic traps [3, 6].

From Eqs. (1) ν_D and ν_Q are expected to scale in the same way with the optical potential depth. In Fig. 2 we report the quadrupole mode frequency as a function of the dipole mode frequency varying the optical lattice height up to 4.1 E_r. Both the dipole and the quadrupole frequency exhibit a strong dependence on the lattice potential (we observed a variation of $\sim$ 30%) demonstrating

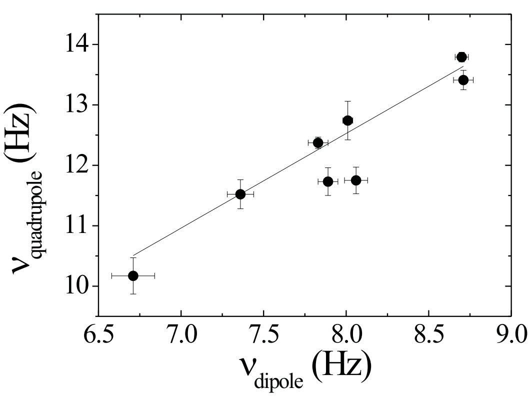

FIG. 2: Frequency of the quadrupole mode of a condensate trapped in the combined potential (harmonic magnetic trap + 1D optical lattice) as a function of the dipole mode frequency measured for different values of the optical lattice depth from 0 E_r to 4.1 E_r. The frequencies of both the modes, characterized by a dynamics along the optical lattice, show a marked dependence from the optical potential depth. The line represent a linear fit with a slope of 1.57 ± 0.01.

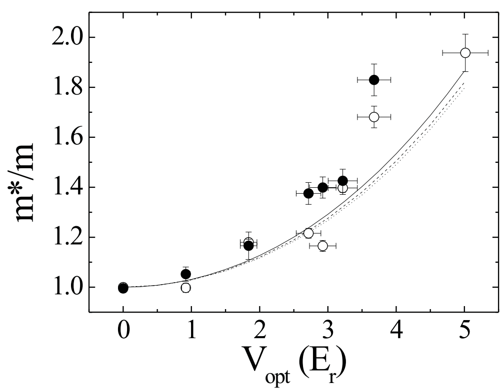

FIG. 3: Effective mass values extracted from the dipole mode frequency (open circle) and from the quadrupole mode frequency (closed circle) as a function of the optical lattice height. The continuous line represent the theoretical curve from [16] obtained neglecting the role of the mean field interactions, while dashed and dotted lines corresponds to the values obtained in [22] numerically solving the Gross-Pitaevskii equation and evaluating the effective mass from the quadrupole and the dipole mode frequencies.

the marked effect of the optical lattice on these modes. Furthermore, from a linear fit of the data shown in Fig. 2 we obtain a slope of 1.57 ± 0.01 in very good agreement with the theoretical prediction of $\sqrt{5/2} = 1.58$. Using Eqs. (1), from our data we can also extract the value of the effective mass m^* as a function of V_{opt}. The results obtained from both the dipole mode and the quadrupole mode frequencies are reported in Fig. 3, together with the theoretical predictions reported in [16] (continuous line). Even if this theoretical curves have been obtained neglecting the mean field interaction and the magnetic confinement, the agreement with our data is very good. In fact, in the regime of V_{opt} explored in our experiment, the effect of interactions is negligible [21] as also confirmed by the direct solution of the Gross-Pitaevskii equation (dashed and dotted line in Fig. 3) [22]. It would be interesting to investigate also the regime of higher optical lattice depth where the effective mass grows exponentially. On the other hand, accessing this regime without entering instability regions seems to be very difficult [20].

Transverse Breathing Mode. In a different series of experiments we also excite the transverse breathing mode modulating the bias field at a frequency close to 2 $\nu_\perp$. In this case the excitation procedure takes only ~ 30 ms and no evident heating of the condensate is observed (from the condensed fraction of the cloud we can estimate a temperature $T < 0.8\ T_c$). We repeat this measurement for different optical lattice depths and the results are summarized in Fig. 4. The experimental data are in agreement with the expected value $2\nu_\perp$ and no dependence on the optical potential height is observed. This confirms the prediction that the dynamical behaviour of

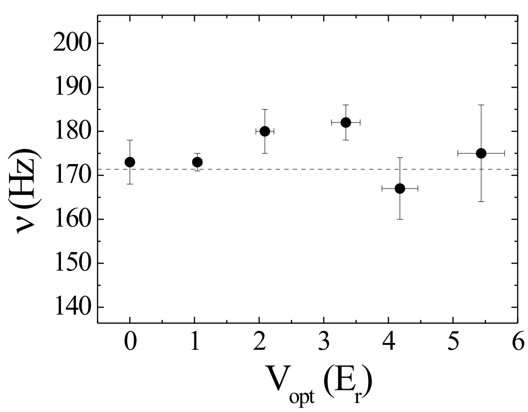

FIG. 4: Frequency of the transverse breathing mode of the condensate in the combined trap as a function of the optical lattice depth V_{opt}. The dashed line corresponds to the expected value $2\nu_\perp$.

the condensate along the radial direction (perpendicular to the lattice axis) is not affected by the presence of the optical potential.

In conclusion, we have investigated the quadrupole and transverse breathing modes of a harmonically trapped condensate in the presence of a 1D optical lattice along the axial direction. The frequency of the quadrupole mode, mainly characterized by an axial oscillation of the

cloud shape, shows an evident dependence on the lattice height in agreement with the renormalized mass theory. On the contrary, the transverse breathing mode, which in our geometry predominantly occurs perpendicularly to the lattice axis, does not exhibit any dependence on the lattice intensity. Our measurements demonstrate that the transport properties of a trapped BEC in the presence of a periodic potential can be described generalizing the hydrodynamic equations of superfluids for a weakly interacting Bose gas. From the measured frequency of the quadrupole and dipole modes we extracted a value for the effective mass that is in very good agreement with the predictions obtained even neglecting in the calculation the effect of interactions.

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[1] D. S. Jin, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **77**, 420 (1996); M. R. Matthews, D. S. Hall, D. S. Jin, J. R. Ensher, C. E. Wieman, E. A. Cornell, F. Dalfovo, C. Minniti, and S. Stringari, Phys. Rev. Lett. **81**, 243 (1998).
[2] M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. M. Kurn, D. S. Durfee, C. G. Townsend, and W. Ketterle, Phys. Rev. Lett. **77**, 988 (1996).
[3] C. Fort, M Prevedelli, F. Minardi, F. S. Cataliotti, L. Ricci, G. M. Tino, and M. Inguscio, Europhys. Lett. **49**, 8 (2000).
[4] O. M. Maragò, S. A. Hopkins, J. Arlt, E. Hodby, G. Hechenblaikner, and C. J. Foot, Phys. Rev. Lett. **84**, 2056 (2000).
[5] D. S. Jin , M. R. Matthews, J. R. Ensher, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **78**, 764 (1997).
[6] D. M. Stamper-Kurn, H.-J. Miesner, S. Inouye, M. R. Andrews, and W. Ketterle, Phys. Rev. Lett. **81**, 500 (1998).
[7] O. Maragò, G. Hechenblaikner, E. Hodby, and C. Foot, Phys. Rev. Lett. **86**, 3938 (2001).
[8] F. Chevy, V. Bretin, P. Rosenbusch, K. W. Madison, and J. Dalibard, Phys. Rev. Lett. **88**, 250402 (2002).
[9] B. P. Anderson and M. A. Kasevich, Science **281**, 1686 (1998).
[10] C. Orzel, A. K. Tuchman, M. L. Fenselau, M. Yasuda, and M. A. Kasevich, Science **291**, 2386 (2001).
[11] F. S. Cataliotti, S. Burger, C. Fort, P. Maddaloni, F. Minardi, A. Trombettoni, A. Smerzi, and M. Inguscio, Science **293**, 843 (2001).
[12] O. Morsch, J. H. Müller, M. Cristiani, D. Ciampini, and E. Arimondo, Phys. Rev. Lett. **87**, 140402 (2001).
[13] M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch and I. Bloch, Nature **415**, 39 (2002).
[14] J. H. Denschlag, J. E. Simsarian, H. Häffner, C. McKenzie, A. Browaeys, D. Cho, K. Helmerson, S. L. Rolston, and W. D. Phillips, J. Phys. B At. Mol. Opt. Phys. **35**, 3095 (2002).
[15] M. Greiner, O. Mandel, T. W. Hänsch, and I. Bloch, Nature **419**, 51 (2002).
[16] M. Krämer, L. Pitaevskii, and S. Stringari, Phys. Rev. Lett. **88**, 180404 (2002).
[17] S. Stringari, Phys. Rev. Lett. **77**, 2360 (1996).
[18] F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **71**, 463 (1999).
[19] P. Pedri, L. Pitaevskii, S. Stringari, C. Fort, S. Burger, F. S. Cataliotti, P. Maddaloni, F. Minardi, M. Inguscio, Phys. Rev. Lett. **87**, 220401 (2001).
[20] F. S. Cataliotti, L. Fallani, F. Ferlaino, C. Fort, P. Maddaloni, M. Inguscio, A. Smerzi, A. Trombettoni, P. G. Kevrekidis, A. R. Bishop, cond-mat/0207139.
[21] M. Krämer private communication.
[22] P. Massignan, and M. Modugno, cond-mat/0205516.